Love Waves in Functionally Graded Piezoelectric Material Structures Loaded with Viscous Liquid

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***Abstract*—We investigate the properties of Love wave propagation in layered functionally graded piezoelectric material(FGPM) structures loaded with viscous liquid. The piezoelectric material is polarized in the z-direction and the material properties change gradually along the thickness of the layer. We here assume that the liquid is conductive. The solutions of dispersion relations are obtained by means of transform matrix method. The effects of the gradient variation of material constants on the phase velocity and attenuation are presented and discussed.**

I. Introduction

Since White invented the interdigital transducers (IDTs) utilized for transmitting and receiving surface acoustic wave (SAW) signals in 1965, SAW has been applied successfully to the electronic industry with filters, delay lines, resonators, and oscillators for signal processing[1]. As we know, a new-style material called functionally graded material (FGM) was proposed to solve problems in the thermal-protection systems of aerospace structures in 1980s. Since then, FGM has attracted interest of investigators from many engineering disciplines. Today, functionally graded piezoelectric materials (FGPMs) can be manufactured and used in SAW devices to improve the efficiency and other features. Hence, the research of wave propagation behaviors and characteristics in FGPM has become a topic of practical importance[2-5]. The authors investigated analytically Love wave propagation in FGPM layer in which the distribution of material properties are assumed to be the same exponential function[6].

As we know, SAW devices may be immersed in a viscous liquid in many sensor applications. Attenuation of the surface acoustic wave modes and leakage of energy into the liquid have to be considered in designing appropriate devices. The development of micro-acoustic wave sensor in biosensing or chemical sensing created the need for further investigations of the surface wave propagation in a layered medium loaded with viscous liquid. A number of acoustic wave modes have been utilized for various sensor applications. The influence of a viscous liquid on acoustic waves propagating in elastic or piezoelectric materials has been studied by many researchers, which is of particular interest for development of liquid sensors[7-9].

Through extensive studies, it is well known that Love wave and SH surface acoustic wave(SH-SAW) liquid sensors are remarkable microacoustic devices with high sensitivity, due to the acoustic energy concentration within a few wavelengths near the surface. Such devices are particularly useful for the measurement of density, viscosity, and acoustic-electric properties of liquids, or electric potential, magnetic potential of applied fields. The layered structures with inhomogeneous boundary conditions, for example, a thin film on a substrate, are currently adopted to achieve high performance for these devices. Numerous investigations have been undertaken for the characteristic analysis of Love waves in layered piezoelectric structures by researchers in various disciplines because of its important applications[10-11]. But so far the investigation of Love waves in FGPM structures loaded with viscous liquid has not been reported.

II. Problem Formulation

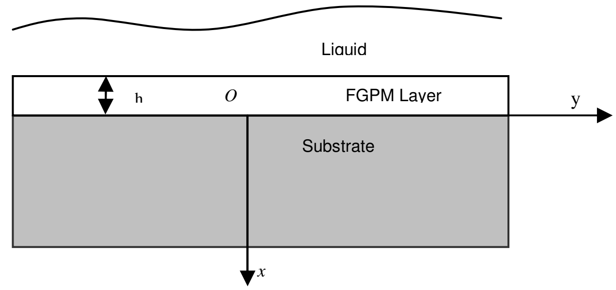

Figure 1. FGPM structure loaded with liquid

The layered FGPM structure loaded with viscous liquid illustrated as Figure1, where a half-space elastic substrate is covered by a thin FGPM layer loaded with viscous liquid. The piezoelectric material is polarized along the z -direction. In order to use transfer matrix method, the FGPM layer is divided into n sublayers, and the thickness of each sublayer is far less than the wavelength of the surface acoustic wave. The thickness of the ith sublayer is h_i . Each sublayer can be regarded as homogenous because of the very small thickness. Assuming the antiplane waves propagate along y -direction, the displacement components and the electric potential are assumed as

$$u = v = 0,\ w = w(x, y, t),\ \phi = \phi(x, y, t), \tag{1}$$

where u, v and w are the displacement components in the x, y and z direction, respectively; ϕ is the electric potential. The coupled wave equations and the constitutive equations for piezoelectric sublayer can be reduced to

$$c_{44}\nabla^2 w + e_{15}\nabla^2\phi = \rho\ddot{w},\ e_{15}\nabla^2 w = \varepsilon_{11}\nabla^2\phi, \tag{2}$$

$$\begin{aligned} T_{xz} &= c_{44}w_{,x} + e_{15}\phi_{,x}, T_{zy} = c_{44}w_{,y} + e_{15}\phi_{,y}, \\ D_x &= e_{15}w_{,x} - \varepsilon_{11}\phi_{,x}, D_y = e_{15}w_{,y} - \varepsilon_{11}\phi_{,y}, \end{aligned} \tag{3}$$

where T_{ij} is the stress tensor, D_i is the electric flux density vector(electric displacement). ρ is the mass density. c_{ij}, e_{ij} and ε_{ij} are the elastic, piezoelectric and dielectric permittivity coefficients, respectively. $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is the two-dimensional Laplace operator in the Cartesian coordinates. The subscript comma denotes a partial derivative with respect to the coordinates and a superimposed dot represents the derivative with respect to time. The governing equations for liquid are simplified to

$$\mu^{\mathrm{L}}\nabla^2 v^{\mathrm{L}} = \rho^{\mathrm{L}}\dot{v}^{\mathrm{L}},\ \nabla^2\phi^{\mathrm{L}} = 0, \tag{4}$$

where ρ^{L} is the mass density of liquid, μ^{L} is the dynamic viscous coefficient of the liquid and v^{L} is the liquid particle velocity in the z-direction. If the liquid is conductive, the boundary conditions at the interface between piezoelectric layer and the liquid can be given as

$$\begin{aligned} \dot{w}(-h, y, t) &= v^{\mathrm{L}}(-h, y, t), \quad \tau_{xz}(-h, y, t) = \tau_{xz}^{\mathrm{L}}(-h, y, t), \\ \phi(-h, y, t) &= \phi^{\mathrm{L}}(-h, y, t), \quad D_x(-h, y, t) = D_x^{\mathrm{L}}(-h, y, t). \end{aligned} \tag{5}$$

The continuous conditions at the interface between the piezoelectric layer and the elastic substrate are expressed as

$$\begin{aligned} w(0, y, t) &= w^{\mathrm{m}}(0, y, t), \quad T_{xz}(0, y, t) = T_{xz}^{\mathrm{m}}(0, y, t), \\ \phi(0, y, t) &= \phi^{\mathrm{m}}(0, y, t), \quad D_x(0, y, t) = D_x^{\mathrm{m}}(0, y, t). \end{aligned} \tag{6}$$

The continuous conditions across the sublayers are

$$\begin{aligned} w^j(x_j, y) &= w^{j+1}(x_j, y), \tau_{xz}^j(x_j, y) = \tau_{xz}^{j+1}(x_j, y), \\ \phi^j(x_j, y) &= \phi^{j+1}(x_j, y), D_x^j(x_j, y) = D_x^{j+1}(x_j, y), \end{aligned} \tag{7}$$

The displacement and electric potential in the substrate tend to zero far from the layer in the positive x-direction

$$x \to +\infty,\ w^m = 0,\ \phi^m = 0.$$

The other radiation conditon is that the velocity and electric potential in the liquid tend to zero far from the layer in the negative x-direction.

III. Solutions of the Problem

We here assume the graded piezoelectric coefficient varying as following function

$$c_{44} = c_{44}^0(1 - \beta x)^2, \tag{8}$$

The solutions in the jth sublayer are expressed as

$$\begin{aligned} w^j &= w^j(x)\exp\left[\mathrm{i}\xi(y - ct)\right], \\ \phi^j &= \phi^j(x)\exp\left[\mathrm{i}\xi(y - ct)\right], \end{aligned} \quad \left(x \in \left[-(j-1)h/n, -jh/n\right]\right),$$

where ξ is the wave number and $\xi = \frac{\omega}{c}(1 + \gamma\mathrm{i}) = k(1 + \gamma\mathrm{i})$ · c and k are the phase velocity and the real part of wave number in the y direction, respectively. γ is the nondimensional attenuation coefficient, and ω is the angular frequency. Substituting it into (2), we can obtain

$$\begin{aligned} & w^{j\prime\prime}(x) + \xi^2(b_1^j)^2 w^j(x) = 0, \\ & \psi^{j\prime\prime}(x) - \xi^2\psi^j(x) = 0, \end{aligned} \tag{9}$$

where $(b_1^j)^2 = \frac{\rho c^2}{c_{44}^*} - 1$, c_{44}^* is given as $c_{44}^* = c_{44}^j + \frac{(e_{15}^j)^2}{\varepsilon_{11}}$, and superscript j indicates the quantities in the jth sublayer. From Eq.(8) we can obtain

$$c_{44}^j = c_{44}^0(1 - \beta x)^2, e_{15}^j = e_{15}^0, (x = -jh/n), \tag{10}$$

for graded piezoelectric coefficient. Then the displacement and electric potential in the jth sublay can be obtained as

$$\begin{aligned} w^j(x, y, t) &= \left[C_1^j\cos(\xi b_1^j x) + C_2^j\sin(\xi b_1^j x)\right]\exp\left[\mathrm{i}\xi(y - ct)\right], \\ \phi^j(x, y, t) &= \left[C_3^j\mathrm{e}^{\xi x} + C_4^j\mathrm{e}^{-\xi x} + \frac{e_{15}}{\varepsilon_{11}}\left(C_1^j\cos(\xi b_1^j x) + C_2^j\sin(\xi b_1^j x)\right)\right] \\ &\quad \cdot\exp\left[\mathrm{i}\xi(y - ct)\right], \end{aligned} \tag{11}$$

The solutions of the displacement and the electric potential of waves in the substrate are given as

$$\begin{aligned} w^{\mathrm{m}} &= f^{\mathrm{m}}(x)\exp\left[\mathrm{i}\xi(y - ct)\right] = B_1^{\mathrm{m}}\mathrm{e}^{-\xi b^{\mathrm{m}} x}\exp\left[\mathrm{i}\xi(y - ct)\right], \\ \phi^{\mathrm{m}} &= B_2^{\mathrm{m}}\mathrm{e}^{-\xi x}\exp\left[\mathrm{i}\xi(y - ct)\right]. \end{aligned} \tag{12}$$

We consider the following solutions of Eqs. (4)

$$\begin{aligned} v^{\mathrm{L}} &= v^{\mathrm{L}}(x)\exp\left[\mathrm{i}\xi(y - ct)\right], \\ \phi^{\mathrm{L}} &= \phi^{\mathrm{L}}(x)\exp\left[\mathrm{i}\xi(y - ct)\right]. \end{aligned} \tag{13}$$

We can obtain

$$v^{\mathrm{L}}(x) = D_1\mathrm{e}^{\lambda x}, \phi^{\mathrm{L}}(x) = D_2\mathrm{e}^{\xi x}, \tag{14}$$

where $\lambda^2 = \xi^2 - \frac{\mathrm{i}\omega\rho^L}{\mu^L}$, $\mathrm{Re}(\lambda) > 0$. D_1 and D_2 are unknown constants to be determined.

IV. Phase Velocity Equations

Defining the vector $\mathbf{C}^j = \left\{C_1^j, C_2^j, C_3^j, C_4^j\right\}^T$, and we can obtain the relation of unknown constants to be determined between the jth and $(j+1)$th sublayer

$$\mathbf{A}_j\mathbf{C}^j = \mathbf{B}_j\mathbf{C}^{j+1}, \tag{15}$$

Furthermore, which can be rewritten as

$$\mathbf{C}^{j+1} = \mathbf{B}_j^{-1}\mathbf{A}_j\mathbf{C}^j, \tag{16}$$

where $\mathbf{B}_j^{-1}$ is the inverse matrix of $\mathbf{B}_j$. Defining the vector

$$\mathbf{D}_j = \mathbf{B}_j^{-1}\mathbf{A}_j, \tag{17}$$

and it is obvious that $\mathbf{A}_j = \mathbf{B}_j$ for homogenous materials, so we can arrive at

$$\mathbf{C}^n = \overbrace{\mathbf{D}_{n-1}\mathbf{D}_{n-2}\cdots\cdots\mathbf{D}_2\mathbf{D}_1}^{n-1}\mathbf{C}^1. \tag{18}$$

From the conditions at interface Eqs.(6) we can obtain

$$\begin{aligned}
&C_1^1 - B_1^{\mathrm{m}} = 0,\\
&C_1^1 b_1^1 (c_{44}^1 + b e_{15}^1 - \mathrm{i}\eta_{44}\omega) + C_3^1 e_{15}^1 - C_4^1 e_{15}^1 = 0,\\
&b C_1^1 + C_3^1 + C_4^1 - B_2^{\mathrm{m}} = 0,\\
&C_2^1 b_1 (e_{15}^1 - b\varepsilon_{11}) - C_3^1 \varepsilon_{11} + C_4^1 \varepsilon_{11} - B_2^{\mathrm{m}} \varepsilon_{11}^{\mathrm{m}} = 0.
\end{aligned} \tag{19}$$

Form the continuous conditions (5), we can obtain the follows

$$\mathrm{i}\omega\cos(b_1^n h\xi) C_1^n + \mathrm{i}\omega\sin(b_1^n h\xi) C_2^n + D_1\exp(-h\lambda) = 0, \tag{20a}$$

$$\begin{aligned}
&C_1^n b_1^n \xi (c_{44}^n + (e_{15}^n)^2 / \varepsilon_{11}) \sin(b_1^n \xi h)\\
&+ C_2^n b_1^n \xi (c_{44}^n + (e_{15}^n)^2 / \varepsilon_{11}) \cos(b_1^n \xi h) +\\
&C_3^n e_{15}^n \xi \exp(-\xi h) - C_4^n e_{15}^n \xi \exp(\xi h) - \mu^{\mathrm{L}}\lambda\exp(-h\lambda) = 0,
\end{aligned} \tag{20b}$$

$$\begin{aligned}
&C_1^n \frac{e_{15}^n}{\varepsilon_{11}} \cos(b_1^n h\xi) - C_2^n \frac{e_{15}^n}{\varepsilon_{11}} \sin(b_1^n h\xi) + C_3^n \exp(-h\xi) +\\
&C_4^n \exp(h\xi) - D_2 \exp(-h\lambda) = 0,
\end{aligned} \tag{20c}$$

$$-C_3^n \varepsilon_{11} \exp(-h\xi) + C_4^n \varepsilon_{11} \exp(h\xi) + D_2 \varepsilon_{11}^{\mathrm{L}} \exp(-h\lambda) = 0. \tag{20d}$$

By means of equations(18), we can rewrite equations (19) and (20) as linearly algebraic equations about $C_1^1, C_2^1, C_3^1, C_4^1, D_1, D_2, B_1^{\mathrm{m}}, B_2^{\mathrm{m}}$. In order to obtain the nontrivial solutions of the above-mentioned unknown constants, the determinant of the coefficient matrix of these linearly algebraic equations must equal zero. So the dispersive relations for the conductive liquid case can be obtained.

V. RESULTS AND DISCUSSION

The material constants of FGPM layer and substrate are given as Table Ⅰ and Ⅱ. The thickness of the FGPM layer is $h = 0.1\mathrm{mm}$.

Fig.2 shows the phase velocity of the first mode for graded elastic modulus for $\mu^{\mathrm{L}} = 0.5\mathrm{N\cdot s/m^2}$. We can find the phase velocity increases with the graded factor.

Fig.3 illustrates the relation between the attenuation and the frequency. We can find that the attenuation increases with the frequency, and decreases with the increase of the graded factor.

Because the phase velocity is dispersive, we should discuss the effect of viscosity of liquid on phase velocity for the certain of wavenumber. Fig.4 presents the phase velocity versus viscous coefficient for electrically open case with graded elastic modulus for k=15000. We can find the phase velocity decreases with increase of the viscosity.

VI. CONCLUSIONS

We investigate the properties of Love wave propagation in layered functionally graded piezoelectric material(FGPM) structures loaded with viscous liquid. The solutions of dispersion relations are obtained by means of transfer matrix method. We can find that attenuation increases with the frequency, and decreases with the increase of the graded factor. It also can be seen that the phase velocity decreases with increase of the viscosity.

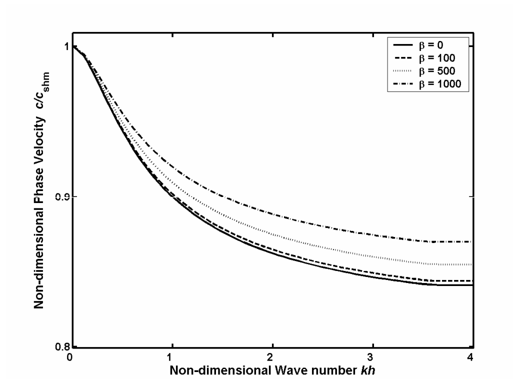

Figure 2. Phase velocity versus wave number

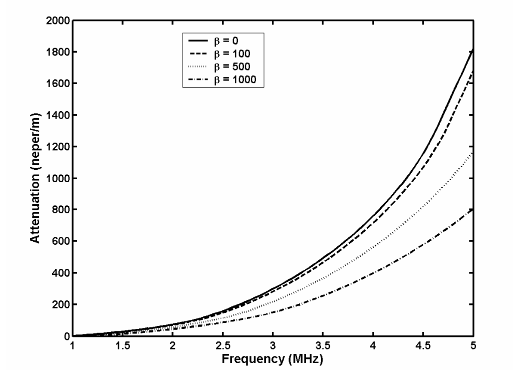

Figure 3. Attenuation versus frequency

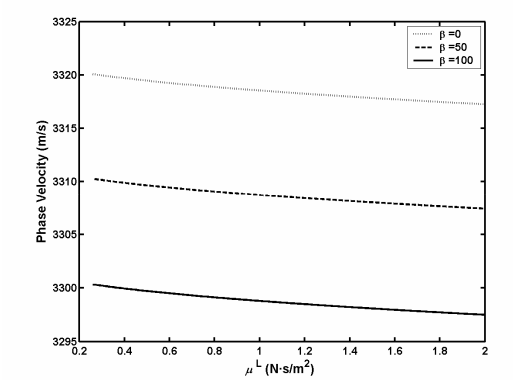

Figure 4. Phase velocity versus viscosity

TABLE I. MATERIAL COEFFICIENTS OF THE PIEZOELECTRIC $BaTiO_3$

c_{44}^{0} $(10^9\,\mathrm{N/m^2})$	e_{15}^{0} $(\mathrm{C/m^2})$	ε_{11} $(10^{-9}\,\mathrm{C^2/Nm^2})$	$\rho(10^3\,\mathrm{kg/m^3})$
43	11.6	11.2	5.8

TABLE II. MATERIAL COEFFICIENTS OF THE SiO_2

c_{44}^{m} $(10^9\,\mathrm{N/m^2})$	ε_{11}^{m} $(10^{-9}\,\mathrm{C^2/Nm^2})$	$\rho^{m}(10^3\,\mathrm{kg/m^3})$
31.2	3.36	2.2

ACKNOWLEDGMENT

The authors gratefully acknowledge the supports of the National Natural Science Foundation of China (No.10772087), K.C.Wong Education Foundation, Hong Kong, and Key Industrial Project of Ningbo (2005B100015).

REFERENCES

[1] R.M. White, F.M. Voltmer, “Direct piezoelectric coupling to surface elastic waves,” Applied Physics Letters, vol. 7, pp.314-315, 1965.

[2] W. Liang, Y.P. Shen, “Gradient surface ply model of SH wave-propagation in SAW sensor,” *ACTA MECHANICA SINICA,* vol. 15(2) 155-163, 1999.

[3] J. Liu, Z.K. Wang, “The propagation behavior of Love waves in a functionally graded layered piezoelectric structure,” *Smart Materials and Structures,* vol. 14, pp.137–46, 2005.

[4] J. Liu, X.S. Gao, Z.K. Wang, “Propagation of Love waves in a smart functionally graded piezoelectric composite structure,” *Smart Materials and Structures,* vol. 16, pp.13-24, 2007.

[5] X.Y. Li, Z.K. Wang, S.H. Huang, “Love waves in functionally graded piezoelectric materials,” *International Journal of Solids and Structures,* vol. 41, pp.7309-7328, 2004.

[6] J.K. Du, X.Y. Jin, J. Wang, K. Xian, “Love wave propagation in functionally graded piezoelectric material layer,” *Ultrasonics, vol.* 46 pp.13-22, 2007.

[7] B.D. Zaitsev, I.E. Kuznetsova, S.G. Joshi, I.A. Borodina, “Acoustic waves in piezoelectric plates bordered with viscous and conductive liquid,” *Ultrasonics,* vol. 39(1), pp.45-50, 2001.

[8] F.L. Guo, R. Sun, “Propagation of Bleustein-Gulyaev wave in 6 mm piezoelectric materials loaded with viscous liquid,” *International Journal of Solids and Structures,* vol. 45(13), pp.3699-3710, 2008.

[9] C. Zhang, J.J. Caron, J.F. Vetelino, “The Bleustein-Gulyaev wave for liquid sensing applications,” *Sensors and Actuators, B: Chemical,* vol. 76(1-3), pp.64-68, 2001

[10] C. McMullan, H. Mehta, E. Gizeli, C.R. Lowe, “Modeling of the mass sensitivity of the Love wave device in the presence of a viscous liquid,” *Journal of Physics D: Applied Physics,* vol. 33, pp.3053-3059, 2000.

[11] L.L. Ke, Y.S. Wang, Z.M. Zhang, “Love waves in an inhomogeneous fluid saturated porous layered half-space with linearly varying properties,” *Soil Dynamics and Earthquake Engineering,* vol. 26(6-7), pp.574-581,2006

[12] Y.C. Lee, S.H. Kuo, “Leaky Lamb wave of a piezoelectric plate subjected to conductive fluid loading: Theoretical analysis and numerical calculation,” *Journal of Applied Physics,* vol. 100(7), pp.073519-1-10, 2006.